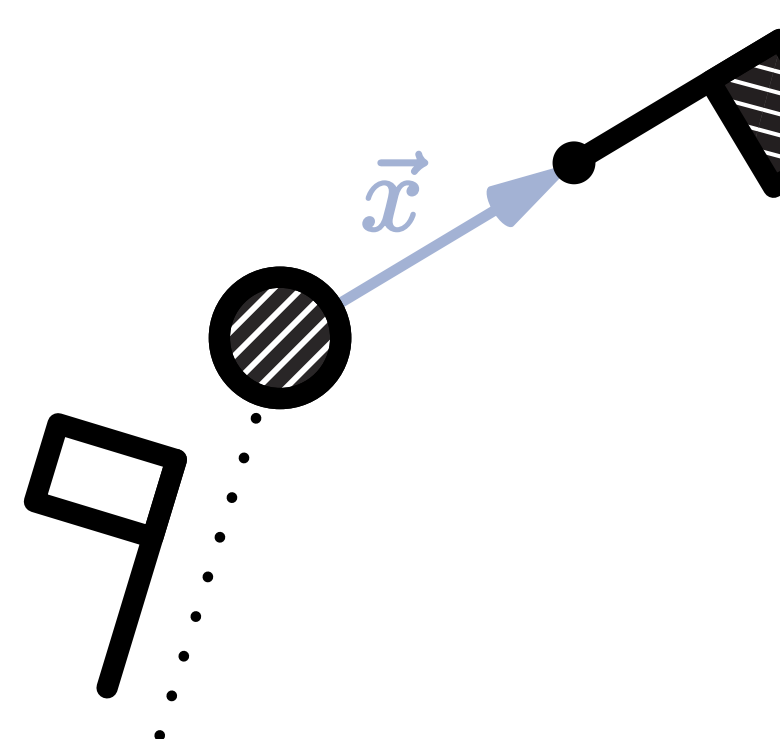


Massive Twistor Theory for Spinning Black Holes

Joonhwi Kim (Caltech)

Newman-Janis Shift

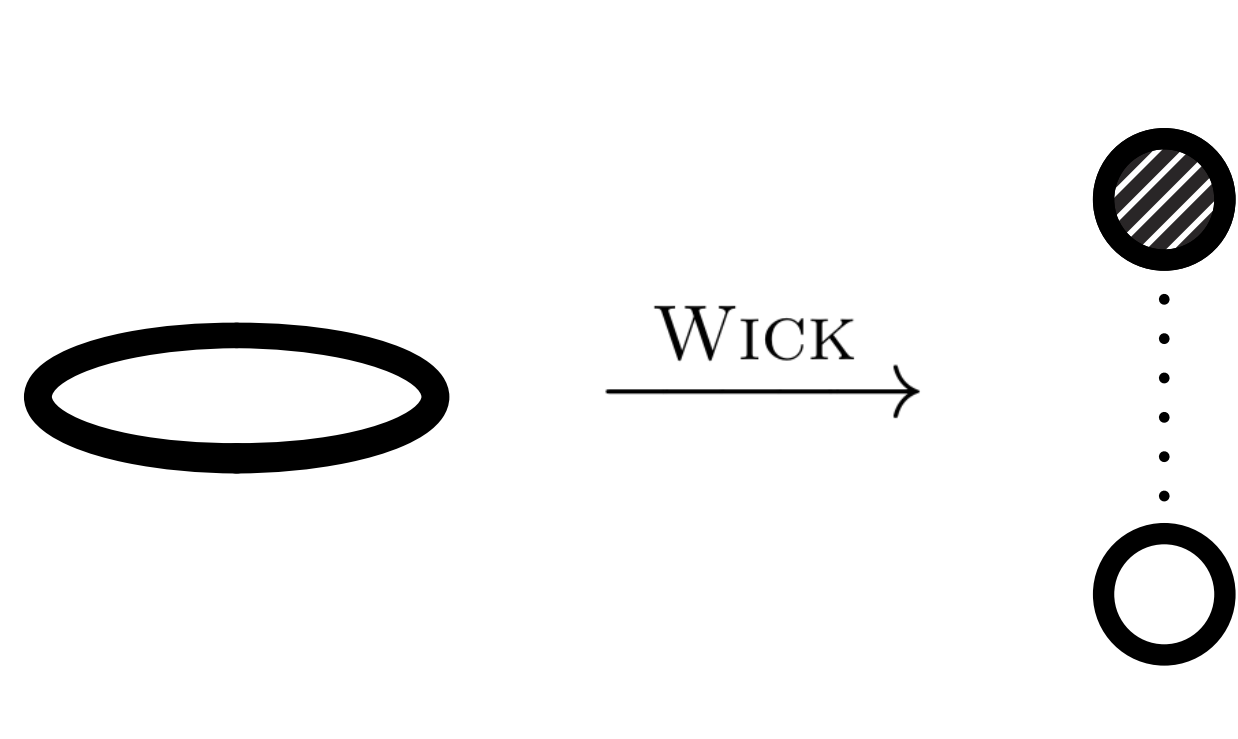
KS metric for SDTN



$$ds_{\text{SDTN}}^2 = ds_{\text{flat}}^2 + \frac{2M}{r} \left(\tilde{o}_{\dot{\alpha}} \eta_{\alpha} dx^{\dot{\alpha}\alpha} \right)^2$$

$$-dt + \frac{x+iy}{r+z} d(x-iy) + dz$$

Kerr = SDTN + ASDTN



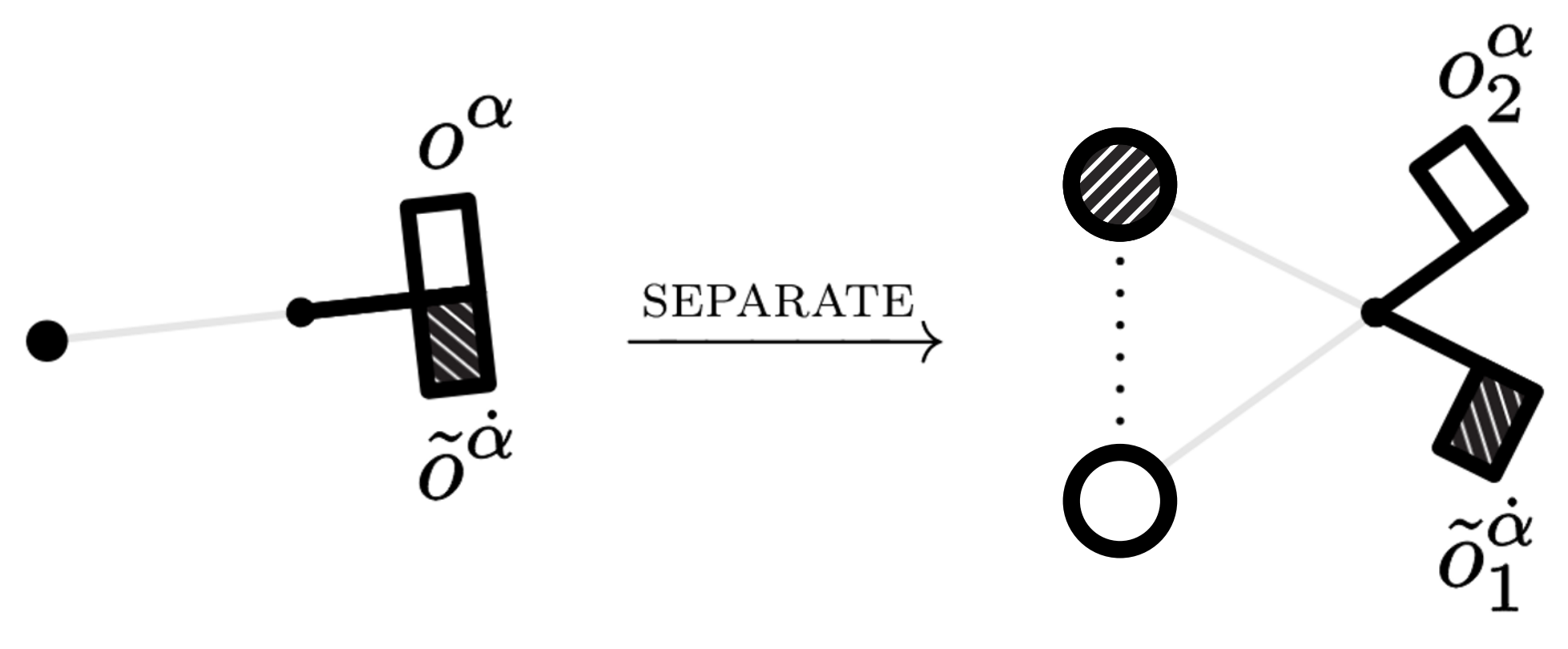
$$(x^2 + y^2 + z^2 - a^2)^2 + (2az)^2 = 0$$

$$(x, y, z) \mapsto (ix, iy, iz)$$

$$(x^2 + y^2 + z^2 + a^2)^2 - (2az)^2 = 0$$

$$(x, y, z) = (0, 0, \pm a)$$

SDTN + ASDTN = Kerr



$$\ell^{\dot{\alpha}\alpha} \propto \tilde{o}_1^{\dot{\alpha}} o_2^{\alpha}$$

"Nonlinear Superposition Theorem"

"BHs in Klein Space" ▶ [2405.09518] [2412.19611] ▶ "Dual Twistor Theory"

Probe Newman-Janis Shift

Heavenly Kerr EoM

Schwarzschild EoM $\dot{x}^{\mu} = p^{\mu} \quad \frac{Dp^{\mu}}{d\tau} = 0$

ASDTN EoM $\dot{z}^{\mu'} = p^{\mu'} \quad \frac{Dp^{\mu'}}{d\tau} = 0 \quad \frac{Dy^{\mu'}}{d\tau} = 0$

Dynamical Newman-Janis Shift at the Probe Level (in Point-Particle EFT) Kerr EoM

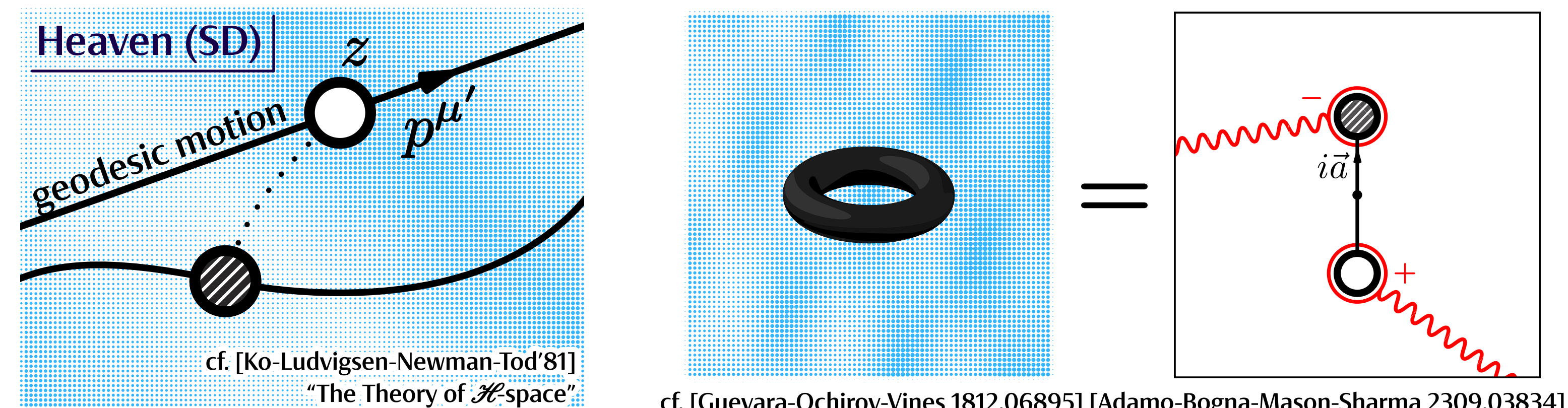
Superintegrability in SDTN Background

Schwarzschild Charges $Q = p_{\mu} K^{\mu}(x)$
 $C = -p_{\mu} Y^{\mu}_{\rho}(x) Y^{\rho}_{\nu}(x) p^{\nu}$

ASDTN Charges $Q = p_{\mu'} K^{\mu'}(z)$
 $C = -p_{\mu'} Y^{\mu'}_{\rho'}(z) Y^{\rho'}_{\nu'}(z) p^{\nu'}$ $R = p_{\mu'} Y^{\mu'}_{\nu'}(z) y^{\nu'}$

Dynamical Newman-Janis Shift of Conserved Charges Kerr Charges
Symmetry Argument for Spin Exponentiation!

Spin Exponentiation = Helicity Selection Rule



"Note on the Kerr Metric" [NJ'65] ▶ "Note on the Kerr EoM" [2512.23697]

Twistor Particle Programme

Massive Twistor Space

Kähler Geometry
 $\{Z_A^I, \bar{Z}_J^B\} = -i\delta_A^B \delta_J^I$

$J^+ \sim \mu \bar{\lambda} \sim (z \wedge p)^+$
 $\sim (x \wedge p)^+ + *(y \wedge p)^+$

Massive Corresp. Space $(z^{\dot{\alpha}\alpha}, \lambda_{\alpha}^I) \cong (x^{\dot{\alpha}\alpha}, y^{\dot{\alpha}\alpha}, \lambda_{\alpha}^I)$

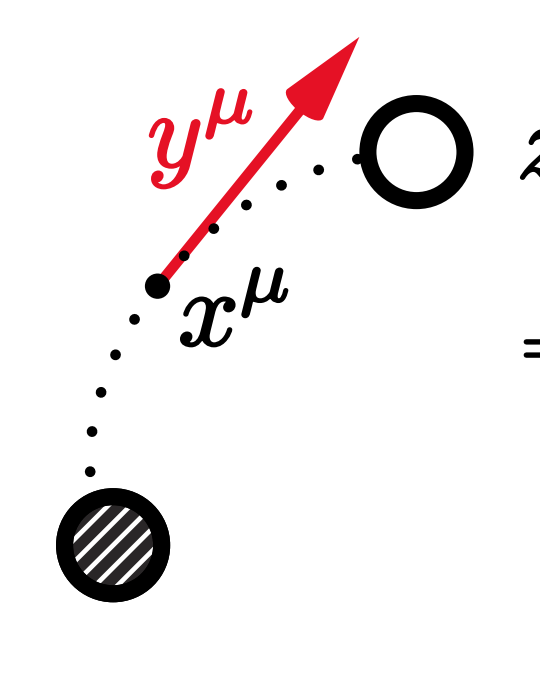
$Z_A^I = (\lambda_{\alpha}^I, \mu^{\dot{\alpha}I})$ $z^{\dot{\alpha}\alpha} = x^{\dot{\alpha}\alpha} + iy^{\dot{\alpha}\alpha}$ $x^{\dot{\alpha}\alpha}$

MT = $(\mathbb{C}^{2,2})^2 \setminus \Delta_0$ $\hat{M} = TM \cong \mathbb{M}^{\mathbb{C}}$ \mathbb{M}

"Double Map"

Curved Spinspacetime $\hat{M} = TM \cong \mathbb{M}^{\mathbb{C}}$

1. Adapted Complex Structure



$$z^{\mu'} = (\text{Exp}^{\mathbb{C}}(x, iy))^{\mu'}$$

$$= \delta^{\mu'}_{\mu} \left(x^{\mu} + iy^{\mu} + \frac{1}{2} \Gamma^{\mu}_{\rho\sigma}(x) y^{\rho} y^{\sigma} + \dots \right)$$

[Hall-Kirwin 0811.3083]
[Burns'93] [GMOOV 2012.11570]

2. Zig-Zag Poisson Brackets

$\{z, z\} = 0$ Preserved by SD Pert.

$\{z^{\dot{\alpha}\alpha}, \bar{z}^{\dot{\beta}\beta}\} = (z - \bar{z})^{\dot{\alpha}\beta} (p^{-1})^{\dot{\beta}\alpha}$

$\{\bar{z}, \bar{z}\} = 0$ Preserved by ASD Pert.

[2309.11886]

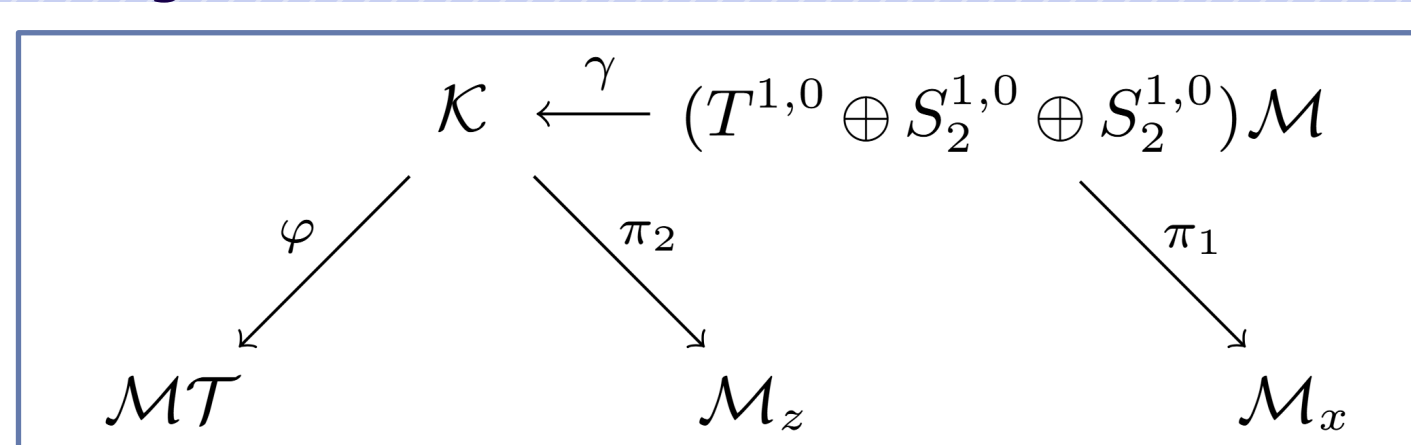
Curved Massive Twistor Theory?

Deformed Massive Incidence Relation?

$$\mu^{\dot{\alpha}I} = z^{\dot{\alpha}\alpha} \lambda_{\alpha}^I$$

$$\tilde{\mu}_I^{\alpha} = \tilde{\lambda}_{I\dot{\alpha}'} (z^{\dot{\alpha}\alpha} - 2iy^{\dot{\alpha}\alpha})$$

Deformation of Symp. Struct. $(\text{MT}, \omega^{\circ}) \rightarrow (\mathcal{MT}, \omega^{\circ} + d\theta_{\text{Kerr}}^{1,0})$



$\lambda_{\alpha}^I, z^{\dot{\alpha}\alpha}$ exhibits Free Motion!

[Penrose'74] [Perjés'75] [Tod'77] [Newman-Winicour'74] ▶ [2602.19495]

From Heaven to Earth

'Googly' Presentation of Kerr Effective Action

$$\theta_{\text{Kerr}} = -\tilde{\lambda}_{I\dot{\alpha}'} e^{\dot{\alpha}\alpha'} \lambda_{\alpha}^I dz^{\mu'} - 2i \tilde{\lambda}_{I\dot{\alpha}'} y^{\dot{\alpha}\alpha'} D\lambda_{\alpha}^I$$

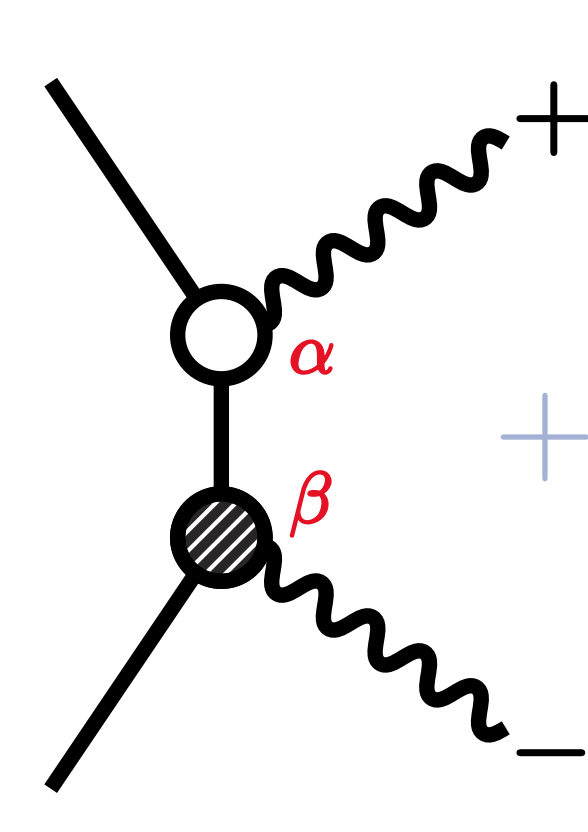
$$+ \sum_{\ell=2}^{\infty} \frac{(-2i)^{\ell}}{\ell!} \sum_{p=1}^{\ell} \sum_{\alpha \in \Omega_p(\ell)} \left[\prod_{i=1}^p \binom{\sum_{j=i}^p \alpha_j - 2}{\alpha_i - 2} \right] \left((Q_{\alpha_1}^-) Q_{\alpha_2} \dots Q_{\alpha_p} \right)^{\mu' \sigma'} dz^{\sigma'}$$

$$+ (\alpha_p - 2) (Q_{\alpha_1}^-) Q_{\alpha_2} \dots Q_{\alpha_{p-1}} Q_{\alpha_p - 1} \right)^{\mu' \sigma'} Dy^{\sigma'}$$

z \tilde{z} Achieve Kerr by Expanding Around ASDTN Worldline where NJ shift (Spin Exponentiation) Manifests!

Kerr Worksheet = Misner String

Kerr Compton Amplitude

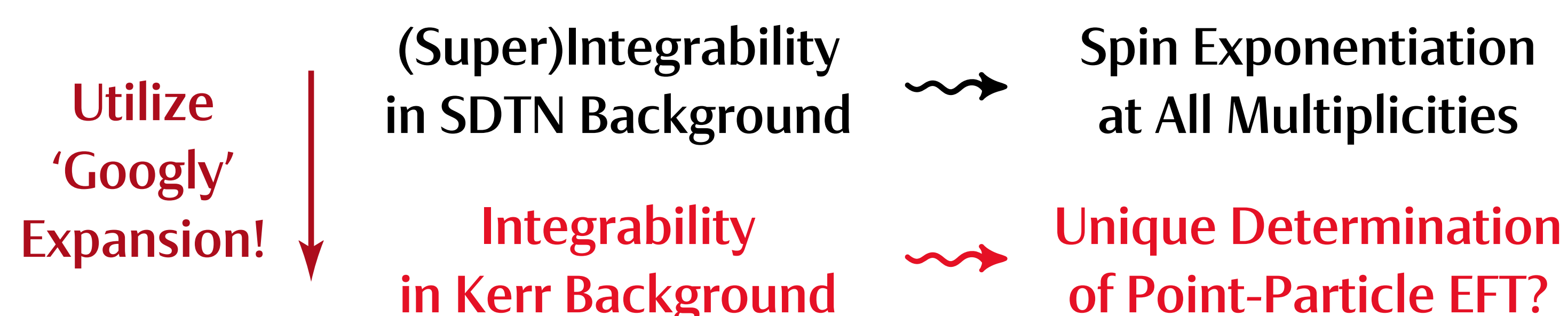


$$\stackrel{\text{YM}}{=} \left[1 + \gamma \left(\frac{\gamma + \beta}{\alpha + \beta} E_1(\alpha) - \frac{\gamma - \alpha}{\alpha + \beta} E_1(-\beta) \right) \right] \mathcal{M}_{\text{Coul}}^{+-} e^{(\beta - \alpha)/2}$$

$$+ \dots \stackrel{\text{GR}}{=} \left[1 + \gamma + \gamma^2 \left(\frac{\gamma + \beta}{\alpha + \beta} E_2(\alpha) - \frac{\gamma - \alpha}{\alpha + \beta} E_2(-\beta) \right) \right] \mathcal{M}_{\text{Sch}}^{+-} e^{(\beta - \alpha)/2}$$

AHH : e^{γ} $E_h(\alpha) := \sum_{\ell=h}^{\infty} \alpha^{\ell-h}/\ell!$

Integrability in Kerr Background?



"A Worksheet for Kerr" ▶ [2602.19495] [2603.07537] [More to Come!]